J. Math. Anal. Appl. 397 (2013) 205–224

Contents lists available at SciVerse ScienceDirect

Journal of Mathematical Analysis and Applications

journal homepage: www.elsevier.com/locate/jmaa

Absolutely continuous multilinear operators

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ARTICLE INFO

Article history:
Received 21 December 2011
Available online 25 July 2012
Submitted by R.M. Aron

Keywords:
Absolutely continuous operator
Multilinear operator
Pietsch domination theorem
Tensor norm
Banach function space
Compact operator

ABSTRACT

We introduce the new class of the $(p; p_1, \dots, p_m; \sigma)$-absolutely continuous multilinear operators, that is defined using a summability property that provides the multilinear version of the (p, σ)-absolutely continuous operators. We give an analogue of the Pietsch domination theorem and a multilinear version of the associated factorization theorem that holds for (p, σ)-absolutely continuous operators, obtaining in this way a rich factorization theory. We present also a tensor norm which represents this multi-ideal by trace duality. As an application, we show that $(p; p_1, \dots, p_m; \sigma)$-absolutely continuous multilinear operators are compact under some requirements. Applications to factorization of linear maps on Banach function spaces through interpolation spaces are also given.

1. Introduction and notation

In 1987 Matter defined the ideal of (p, σ)-absolutely continuous linear operators in order to analyze super-reflexive Banach spaces, establishing many of its fundamental properties in [1]. In the nineties, López Molina and Sánchez Pérez studied the factorization properties and the trace duality for these operators in a series of papers, introducing the class of tensor norms that represent these operator ideals (see [2–4]). Roughly speaking, the class of (p, σ)-absolutely continuous operators can be considered as an "interpolated" ideal between the p-summing operators and the continuous operators, preserving some of the characteristic properties of the first class. Let $1 \le p < \infty$ and $0 \le \sigma < 1$. A linear operator T between Banach spaces X and Y is (p, σ)-absolutely continuous if there is a positive constant C such that for all $n \in \mathbb{N}$, $(x_i)_{i=1}^n \subset X$, we have

$$\left(\sum_{i=1}^{n} \|T(x_i)\|^{\frac{p}{1-\sigma}}\right)^{\frac{1-\sigma}{p}} \le C \sup_{\xi \in B_{X^*}} \left(\sum_{i=1}^{n} \left(|\langle x_i, \xi\rangle|^{1-\sigma} \|x_i\|^{\sigma}\right)^{\frac{p}{1-\sigma}}\right)^{\frac{1-\sigma}{p}}. \tag{1.1}$$

The smallest constant C such that the inequality (1.1) holds is called the (p, σ)-absolutely continuous norm of T, and is denoted by $\pi_{p,\sigma}(T)$. It is in fact a norm on the space $\mathcal{P}_{p,\sigma}(X, Y)$ of all (p, σ)-absolutely continuous linear operators from X into Y, that becomes a Banach space. In particular, we have that $\mathcal{P}_{p,0}(X, Y)$ coincides with $\Pi_p(X, Y)$, the well known operator ideal of p-summing operators introduced by Pietsch in [5] (see also [6,7]).

The aim of this paper is to study the multilinear version of this class of operators and its tensor product representation, and to provide some applications in the setting of the factorization theory of bilinear maps. For more details concerning the nonlinear theory of absolutely summing operators and recent developments and applications we refer to [8–16]. Regarding compactness, we show that as in the case of the p-summing operators the (p, σ)-absolutely continuous operators

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0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2012.07.034