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Factorization of absolutely continuous polynomials

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ABSTRACT

In this paper we study the ideal of dominated $(p; \sigma)$-continuous polynomials, that extend the nowadays well known ideal of p-dominated polynomials to the more general setting of the interpolated ideals of polynomials. We give the polynomial version of Pietsch's factorization Theorem for this new ideal. Although based on Botelho et al. (in press) [16], our factorization theorem requires new techniques inspired in the theory of Banach lattices.

0. Introduction

The operator ideal of (p, σ)-absolutely continuous operators was introduced in 1987 in order to analyze super-reflexivity and some other properties of Banach spaces [37]. This new ideal was created by means of a general interpolation procedure due to Jarchow and Matter [29], and must be understood as an ideal located in between absolutely p-summing operators and continuous operators. Absolutely summing operators are the core of many classes of (linear and non-linear) operator ideals, and the reader is referred to the classical monograph [25] for their study or to textbooks on operator ideals such as [22,49]. Matter [38] applied (p, σ)-absolutely continuous operators to obtain a description of operators factoring through super-reflexive Banach spaces. Later, several authors studied factorization properties of this new class of operators, the tensor product representation and found more applications (see for example [2,32,33]).

The multi-ideal of $(p; p_1, \dots, p_m; \sigma)$-absolutely continuous multilinear operators on Banach spaces has been recently defined and characterized by Dahia et al. in [21] as a natural multilinear extension of the classical ideal of $(p; \sigma)$-absolutely continuous linear operators. This multi-ideal has many good properties and extends almost all the ones that are satisfied by the ideals of absolutely p-summing and p-dominated multilinear operators, as inclusion theorems, Pietsch's domination theorems, factorization theorems and tensor product representations. It is worth mentioning that $(r, 1)$-summing linear operators were extended naturally to multiple summing operators (see [5, Definition 2.1] and [36, Definition 2.2]). Among the applications of multiple summing operators on several areas in mathematics, we highlight the well known Bohnenblust–Hille inequalities, that can be reformulated by saying that each bounded m-linear form defined on Banach spaces is multiple $(\frac{2m}{m+1}, 1)$-summing [5]. The Bohnenblust–Hille inequality and its applications to Bohr's absolute convergence problem on Dirichlet series has yielded to a wide research production on these subjects by several authors such as A. Defant, L. Frerick, G. A. Muñoz-Fernández, D. Núñez-Alarcón, J. Ortega-Cerdá, M. Ounaïes, D. Pellegrino, D. Popa, U. Schwarting, K. Seip, and J. B. Seoane-Sepúlveda among others (see [42] and the references therein). In the remarkable paper [23] it is proved that the Bohnenblust–Hille inequality for complex homogeneous polynomials is hypercontractive. The positive answer to the hypercontractivity of the real case has been proved very recently in [18]. The polynomial and multilinear

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