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MULTILINEAR OPERATORS BETWEEN ASYMMETRIC NORMED SPACES

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Abstract. We introduce and characterize the continuity of multilinear mappings between asymmetric normed spaces. In particular, we study the completeness properties of the asymmetric normed semilinear space of these mappings. As an application, we prove multilinear versions of the Banach–Steinhaus and closed graph theorems in the framework of asymmetric normed spaces.

1. Introduction and preliminaries. The paper is divided into three sections. After the introductory one, in Section 2 we extend to multilinear mappings the concept of continuity in asymmetric normed spaces. In Section 3 we establish some fundamental theorems: separate continuity of multilinear mappings in asymmetric normed spaces, the multilinear asymmetric Banach–Steinhaus theorem and the closed graph theorem for continuous multilinear operators between asymmetric normed spaces.

The notation used in the paper is in general standard. Let $m \in \mathbb{N}$ and let X_j $(j = 1, \ldots, m)$, Y be normed spaces over $\mathbb{K}$ (either $\mathbb{R}$ or $\mathbb{C}$). A mapping $T : X_1 \times \cdots \times X_m \to Y$ is called *multilinear* (or *m-linear*) if it is linear separately in each coordinate, i.e. the mappings

$$T_j : X_j \to Y, \quad x^j \mapsto T(x^1, \ldots, x^j, \ldots, x^m),$$

are linear for any fixed $x^k \in X_k$, $k \neq j$. The linear space of such mappings is denoted by $L(X_1, \ldots, X_m; Y)$.

An m-linear mapping $T : X_1 \times \cdots \times X_m \to Y$ is continuous if it is continuous as a function between two normed spaces. As a consequence, T is continuous if and only if there is a constant $C \geq 0$ such that

$$\|T(x^1, \ldots, x^m)\| \leq C\|x^1\| \cdots \|x^m\| \tag{1.1}$$

for all $x^j \in X_j$ $(j = 1, \ldots, m)$.

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