



## The Lipschitz injective hull of Lipschitz operator ideals and applications

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### Abstract

We introduce and study the Lipschitz injective hull of Lipschitz operator ideals defined between metric spaces. We show some properties and apply the results to the ideal of Lipschitz  $p$ -nuclear operators, obtaining the ideal of Lipschitz quasi  $p$ -nuclear operators. Also, we introduce in a natural way the ideal of Lipschitz Pietsch  $p$ -integral operators and show that its Lipschitz injective hull coincide with the ideal of Lipschitz  $p$ -summing operators defined by Farmer and Johnson. Finally, we consider both ideals as Lipschitz operator ideals between a metric space and a Banach space, showing that these ideals are not of composition type. Their maximal hull and minimal kernel are also studied.

**Keywords** Lipschitz operator ideals · Injective hull of operator ideals · Quasi  $p$ -nuclear operators

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