# Two-Lipschitz operator ideals

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A B S T R A C T

We introduce and investigate the concept of two-Lipschitz operator ideal between pointed metric spaces and Banach spaces. We show the basics of this new theory and we give a procedure to create a two-Lipschitz operator ideal from a linear operator ideal. We apply our result to the ideals of strongly $p$-summing and compact linear operator to obtain their corresponding two-Lipschitz operator ideal. Also, we establish a natural relation between two-Lipschitz and bilinear maps and show that the two-Lipschitz factorable $p$-dominated operators are those which are associated to the well-known $p$-semi-integral bilinear operators.

## 0. Introduction

The theory of operator ideals has proved to be a strong tool for the investigation and classification of linear operators between Banach spaces. Nowadays, it has become the starting point to understand and solve new problems related to non-linear operators. The linear theory has spread to Lipschitz operators, leading to the notions of Lipschitz operator ideals. A first outline of such a Lipschitz theory was given by Farmer and Johnson in 2009 (see [18]). In 2016 an axiomatic theory of Lipschitz operator ideals for Banach spaces-valued Lipschitz mappings was given by Achour et al. in [3] (see also [35]) and in [7] for Lipschitz operator ideals between pointed metric spaces. These new Lipschitz operator ideals could also be considered to be a point for the study of some specific properties of non-linear operators, that can be considered as a new area in non-linear functional analysis.

An ideal of Lipschitz mappings $\mathcal{I}_{Lip}$ is a subclass of the class of all Lipschitz mappings between pointed metric spaces and Banach spaces such that for a metric space $X$ and Banach space $E$, the component

$$\mathcal{I}_{Lip}(X, E) := Lip_0(X, E) \cap \mathcal{I}_{Lip},$$

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