# THE EXTENSION OF TWO-LIPSCHITZ OPERATORS

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Abstract. The paper deals with some further results concerning the class of two-Lipschitz operators. We prove first an isometric isomorphism identification of two-Lipschitz operators and Lipschitz operators. After defining and characterizing the adjoint of a two-Lipschitz operator, we prove a Schauder type theorem on the compactness of the adjoint. We study the extension of two-Lipschitz operators from the cartesian product of two complemented subspaces of a Banach space to the cartesian product of whole spaces. Also, we show that every two-Lipschitz functional defined on the cartesian product of two pointed metric spaces admits an extension with the same two-Lipschitz norm under some requirements on domaine spaces.

## 1. Introduction and notation

Sánchez Pérez in [14] introduced the definition of real two-Lipschitz maps acting in a cartesian product of two pointed metric spaces (called Lipschitz bi-forms) which possess a continuous bi-linearization between Banach spaces. A detailed and systematic study of these mappings with values in a Banach space is given recently in [10], where the authors introduce the new concept of two-Lipschitz operator ideals between pointed metric spaces and Banach spaces. A number of class of linear operators have been fruitfully generalized to the Lipschitz setting in recent years by several authors (see [1], [2], [3], [4], [7] and the references therein). Note that the concept of a two-Lipschitz mapping was firstly introduced by Dubei et al. [9] as those mappings that are Lipschitz in each variable.

In the present paper we go further in this direction and prove some results concerning this class of non-linear mappings. After the introductory one, in Section 2, by using the linearization of Lipschitz operators and the bi-linearization of two-Lipschitz operators, we obtain an important canonical identification of two-Lipschitz operators and Lipschitz operators. We define the adjoint of a two-Lipschitz operator which is the key for proving the Schauder type theorem for compact two-Lipschitz operators. In Section 3, we present some theorems on

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